Sol - 06/10/2023

(1) $ax^2 + by^2 = 2cz$ — (1)

$lx + my + nz = p$ — (2)

Since generators are parallel to z-axis, therefore eliminating z

$$ax^2 + by^2 = 2c\left(\frac{p - lx - my}{n}\right)$$

$$\Rightarrow anx^2 + bny^2 + 2clx + 2cmy = 2cp$$

(Option c)

(2) $x^2+y^2+z^2 = 9$ — (1)

$x - y + z = 3$

The axis of the cylinder is normal to the plane $x-y+z=3$

Hence direction cosines are proportional to $1, -1, 1$

Let (α, β, γ) be any point on the cylinder. Then the eq^n of the generator through this point

$$\frac{x-\alpha}{1} = \frac{y-\beta}{-1} = \frac{z-\gamma}{1} = r \quad \text{— (1)}$$

Any point on the generator is $(r+\alpha,\ \beta-r,\ r+\gamma)$. If this point lies on the guiding curve then

$$(\alpha+r)^2 + (\beta-r)^2 + (r+\gamma)^2 = 9 \quad \text{— (2)}$$

and $(\alpha+r) - (\beta-r) + (r+\gamma) = 3$

$$\alpha + r - \beta + r + r + \gamma = 3$$

$$3r = 3 - \alpha + \beta - \gamma$$

$$r = \frac{3-\alpha+\beta-\gamma}{3}$$

Putting the value of r in (2)

$$\left\{\alpha + \frac{3-\alpha+\beta-\gamma}{3}\right\}^2 + \left\{\beta - \frac{3-\alpha+\beta-\gamma}{3}\right\}^2 + \left\{\gamma + \frac{3-\alpha+\beta-\gamma}{3}\right\}^2 = 9$$

$$\alpha^2+\beta^2+\gamma^2 + \frac{(3-\alpha+\beta-\gamma)^2}{9} + \frac{(3-\alpha+\beta-\gamma)^2}{9} + \left(\frac{3-\alpha+\beta-\gamma}{9}\right)^2 + \frac{2\alpha(3-\alpha+\beta-\gamma)}{3} - \frac{2\beta(3-\alpha+\beta-\gamma)}{3} + \frac{2\gamma(3-\alpha+\beta-\gamma)}{3} = 9$$

$$\alpha^2+\beta^2+\gamma^2 + \frac{(3-\alpha+\beta-\gamma)^2}{3} + \frac{2\alpha(3-\alpha+\beta-\gamma)}{3} + \frac{-2\beta(3-\alpha+\beta-\gamma)}{3} + \frac{2\gamma(3-\alpha+\beta-\gamma)}{3} = 9$$

$$\alpha^2+\beta^2+\gamma^2 + \frac{(3-\alpha+\beta-\gamma)^2}{3} + \frac{(3-\alpha+\beta-\gamma)(2\alpha-2\beta+2\gamma)}{3} = 9$$

$$\alpha^2+\beta^2+\gamma^2 + \frac{(3-\alpha+\beta-\gamma)\left[3-\alpha+\beta-\gamma+2\alpha-2\beta+2\gamma\right]}{3} = 9$$

$$\alpha^2+\beta^2+\gamma^2 + \frac{(3-\alpha+\beta-\gamma)(3+\alpha-\beta+\gamma)}{3} = 9$$

$$3\alpha^2+3\beta^2+3\gamma^2 - \alpha^2-\beta^2-\gamma^2 + 2\alpha\beta - 2\alpha\gamma + 2\beta\gamma + 9 = 27$$

$$2(\alpha^2+\beta^2+\gamma^2 + \beta\gamma - \gamma\alpha + \alpha\beta) = 18$$

$$\alpha^2+\beta^2+\gamma^2 + \beta\gamma - \gamma\alpha + \alpha\beta = 9$$

The locus of (α, β, γ) is

$$\boxed{x^2+y^2+z^2 + yz - zx + xy - 9 = 0}$$

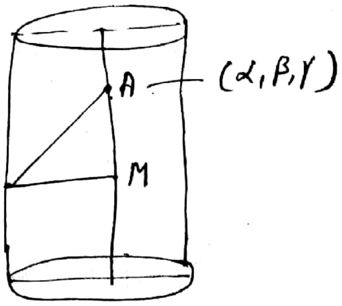
(Option a)

③ It is given that the equation of the axis are

$$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$$

$$\Rightarrow \frac{x-1}{2/3} = \frac{y-2}{1/3} = \frac{z-3}{2/3}$$ and radius of the cylinder is 2

$$AP^2 - AM^2 = r^2$$



$$\left\{(\alpha-1)^2 + (\beta-2)^2 + (\gamma-3)^2\right\}$$

$$- \left\{\frac{2}{3}(\alpha-1) + \frac{1}{3}(\beta-2) + \frac{2}{3}(\gamma-3)\right\}^2 \quad (1,2,3) \leftarrow P$$

$$= 4$$

$$9\left\{(\alpha-1)^2 + (\beta-2)^2 + (\gamma-3)^2\right\} - (2\alpha+\beta+2\gamma-2-2-6)^2 = 36$$

$$9\left\{(\alpha-1)^2 + (\beta-2)^2 + (\gamma-3)^2\right\} - (2\alpha+\beta+2\gamma-10)^2 = 36$$

$$9\left\{\alpha^2+\beta^2+\gamma^2-2\alpha-4\beta-6\gamma+1+4+9\right\} - (2\alpha+\beta+2\gamma-10)^2 = 36$$

$$9\alpha^2 + 9\beta^2 + 9\gamma^2 - 18\alpha - 36\beta - 54\beta + 126 - (4\alpha^2+\beta^2+4\gamma^2$$
$$+ 4\alpha\beta + 4\beta\gamma + 8\alpha\gamma) - 100 + 20(2\alpha+\beta+2\gamma) = 36$$

$$9\alpha^2 + 9\beta^2 + 9\gamma^2 - 18\alpha - 36\beta - 54\beta + 126 - 4\alpha^2 - \beta^2 - 4\gamma^2$$
$$- 4\alpha\beta - 4\beta\gamma - 8\alpha\gamma - 100 + 40\alpha + 20\beta + 40\gamma = 36$$

$$5\alpha^2 + 8\beta^2 + 5\gamma^2 - 4\beta\gamma - 4\alpha\beta - 8\alpha\gamma + 22\alpha - 16\beta - 14\gamma = 10$$

$$5\alpha^2 + 8\beta^2 + 5\gamma^2 - 4\beta\gamma - 4\alpha\beta - 8\alpha\gamma + 22\alpha - 16\beta - 14\gamma - 10 = 0$$

Locus of (α, β, γ) is

$$\boxed{5x^2 + 8y^2 + 5z^2 - 4yz - 4xy - 8xz + 22x - 16y - 14z - 10 = 0}$$

option a

④ The cone passing through the axes will have the vertex at the origin is

$$ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0 \quad \text{— ①}$$

But this cone passes through the coordinate axes. Therefore, the direction cosines of the axes 1, 0, 0, 0, 1, 0, 0, 0, 1 will satisfy the eq^n ①

$$a = b = c = 0$$

The equation of the cone becomes $\boxed{fyz + gzx + hxy = 0}$

Option a

⑤ Let equation of line passing through (α, β, γ) is

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$$

let it be a generator.

It meets the plane $z = 0$ at $\left(\alpha - \frac{l\gamma}{n}, \beta - \frac{m\gamma}{n}, 0\right)$. It must lie on $y^2 = 4ax$

$$\left(\beta - \frac{m\gamma}{n}\right)^2 = 4a\left(\alpha - \frac{l\gamma}{n}\right)$$

$$\left(\beta - \gamma\frac{m}{n}\right)^2 = 4a\left(\alpha - \gamma\frac{l}{n}\right)$$

$$\left[\beta - \left(\frac{y-\beta}{z-\gamma}\right)\gamma\right]^2 = 4a\left[\alpha - \left(\frac{x-\alpha}{z-\gamma}\right)\gamma\right]$$

$$(\beta z - \cancel{\beta\gamma} - y\gamma + \cancel{\beta\gamma})^2$$

$$= 4a(z-\gamma)\left[\alpha z - \cancel{\alpha\gamma} - \gamma x + \cancel{\alpha\gamma}\right]$$

$$(\beta z - \gamma y)^2 = 4a(z-\gamma)(\alpha z - \gamma x)$$

$$\beta^2 z^2 + \gamma^2 y^2 - 2\beta\gamma yz$$

$$= 4a(\alpha z^2 - \gamma zx - \alpha\gamma z + \gamma^2 x)$$

$$\beta^2 z^2 + \gamma^2 y^2 - 2\beta\gamma yz$$

$$= 4a\alpha z^2 - 4a\gamma zx - 4a\alpha\gamma z + 4a\gamma^2 x$$

$$\gamma^2 y^2 + (\beta^2 - 4a\alpha) z^2 - 2\beta\gamma y \cdot z$$

$$+ 4a\gamma \cdot zx - 4a\gamma^2 \cdot x + 4a\gamma\alpha \cdot z = 0$$

Option b

⑥ Let (x', y', z') be the vertex of the cone

$$ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0 \quad \text{— ①}$$

$$a(x+x')^2 + b(y+y')^2 + c(z+z')^2$$

$$+ 2u(x+x') + 2v(y+y') + 2w(z+z') + d = 0$$

If the equation is homogeneous equation then the coefficient of x, y, z and absolute term must vanish.

$$ax' + u = 0, \quad by' + v = 0, \quad cz' + w = 0$$

$$\Rightarrow x' = \frac{-u}{a} \qquad y' = \frac{-v}{b} \qquad z' = \frac{-w}{c}$$

and $ax'^2 + by'^2 + cz'^2$

$+ 2ux' + 2vy' + 2wz' + d = 0$

$$\frac{u^2}{a} + \frac{v^2}{b} + \frac{w^2}{c} - \frac{2u^2}{a} - \frac{2v^2}{b} - \frac{2w^2}{c} + d = 0$$

$$-\frac{u^2}{a} - \frac{v^2}{b} - \frac{w^2}{c} + d = 0$$

$$\boxed{\frac{u^2}{a} + \frac{v^2}{b} + \frac{w^2}{c} = d}$$

Option a

7 The equation of the sphere through the given circle is

$$x^2+y^2+z^2 - a^2 + \lambda z = 0$$

It passes through (α, β, γ) then

$$\alpha^2+\beta^2+\gamma^2 - a^2 + \lambda\gamma = 0$$

$$\lambda = \frac{-(\alpha^2+\beta^2+\gamma^2-a^2)}{\gamma}$$

$$x^2+y^2+z^2 - a^2 + \frac{-(\alpha^2+\beta^2+\gamma^2-a^2)}{\gamma} \cdot z = 0$$

$$\gamma(x^2+y^2+z^2-a^2) = z(\alpha^2+\beta^2+\gamma^2-a^2)$$

Option a

8 Let the equation of the sphere be

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0 \quad \text{— (1)}$$

This sphere is passes through $(a,0,0)$, $(0,b,0)$ and $(0,0,c)$

then

$a^2 + 2ua = 0 \Rightarrow 2u = -a$

$b^2 + 2Vb = 0 \Rightarrow 2V = -b$

$c^2 + 2wC = 0 \Rightarrow 2W = -C$

from (1)

$$\boxed{x^2+y^2+z^2 - ax - by - cz + d = 0}$$

Option b

9

A	—	$\frac{\lfloor 5}{\lfloor 2}$	$= 60$
H	—	$\frac{\lfloor 5}{\lfloor 2 \lfloor 2}$	$= 30$
I	—	$\frac{\lfloor 5}{\lfloor 2 \lfloor 2}$	$= 30$
MAA	—	$\lfloor 3$	$= 6$
MAHA	—	$\lfloor 2$	$= 2$
MAHIA	—	$\lfloor 1$	$= 1$
MAHIMA	—	1	$= 1$
			130

Rank of the word MAHIMA in dictionary $= 130$

Option a

10

G	—	$\lfloor 3$	$= 6$
I	—	$\lfloor 3$	$= 6$
KG	—	$\lfloor 2$	$= 2$
KIG	—	$\lfloor 1$	$= 1$
KING	—	1	$= 1$
			16

Rank of the word KING in dictionary $= 16$ Option a