Sol^n 04/10/2023

① $n(A\cup B) = n(A) + n(B) - n(A\cap B)$

$= 200 + 300 - 100$

$= 400$

$n(A'\cap B') = n(U) - n(A\cup B)$

$= 700 - 400$

$= 300$

option a

② $\begin{bmatrix} 2 & k \\ 3 & 5 \end{bmatrix}$

Inverse of the given matrix will not exist if $|A| = 0$

$10 - 3k = 0$

$\Rightarrow \boxed{k = 10/3}$

option d

③ $\frac{d^4y}{dx^4} + 3\left(\frac{d^2y}{dx^2}\right)^5 + 5y = 0$

order $= 4$

degree $= 1$

④ $\lim_{n\to\infty}\left(\frac{1}{1+\sqrt{n}} + \frac{1}{2+\sqrt{2n}} + \cdots + \frac{1}{n+\sqrt{nn}}\right)$

$= \sum_{r=1}^{n} \frac{1}{r+\sqrt{rn}}$

$= \sum_{r=1}^{n} \frac{1/n}{\frac{r}{n}+\sqrt{\frac{r}{n}}}$

$= \sum_{r=1}^{n} \frac{1}{\frac{r}{n}+\sqrt{\frac{r}{n}}}\cdot\frac{1}{n}$

$= \int_0^1 \frac{1}{x+\sqrt{x}}\,dx$

$= \int_0^1 \frac{1}{\sqrt{x}(1+\sqrt{x})}\,dx$

Let $\sqrt{x} = t$

$\frac{1}{2\sqrt{x}}dx = dt$

$\frac{1}{\sqrt{x}}dx = 2dt$

$= \int_0^1 \frac{2}{t+1}\,dt$

$= 2\log(t+1)\Big|_0^1$

$= 2\log 2$

option a

⑤ given $\bar{a} \neq 0,\ \bar{b} \neq 0$

$|\bar{a}+\bar{b}| = |\bar{a}-\bar{b}|$

$\cancel{|a|^2} + \cancel{|\bar{b}|^2} + 2|a||b|\cos\theta = \cancel{|a|^2} + \cancel{|b|^2} - 2|a||b|\cos\theta$

$4|a||b|\cos\theta = 0$

$|a||b|\cos\theta = 0$

$\boxed{a\cdot b = 0}$

$\Rightarrow a \perp b$ option c

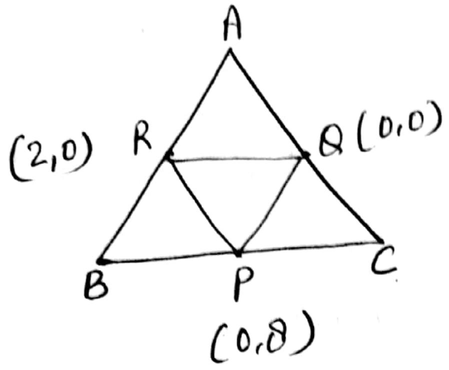
⑥ $x^2 - y^2 - 2y - 1 = 0$

Here $a + b = 0$

$\Rightarrow \boxed{\theta = \pi/2}$

option d

(7)



Area of Δ ABC

$= 4 \cdot$ area PQR

$= \cancel{4}^{2} \cdot \frac{1}{\cancel{2}} \times [16 - 0]$

$= 32$

Option d

0 8
0 0
2 0
0 8

(9) $\int_0^{\pi/2} \sin^6\theta \, d\theta$

$= \int_0^{\pi/2} \cos^0\theta \sin^6\theta \, d\theta$

$= \dfrac{\Gamma\frac{0+1}{2} \, \Gamma\frac{6+1}{2}}{2 \, \Gamma\frac{0+6+2}{2}}$

$= \dfrac{\Gamma\frac{1}{2} \, \Gamma\frac{7}{2}}{2 \, \Gamma 4}$

$= \dfrac{\sqrt{\pi} \cdot \frac{5}{2} \cdot \frac{\cancel{3}}{2} \cdot \frac{1}{2} \sqrt{\pi}}{2 \cdot \cancel{3} \cdot 2 \cdot 1}$

$= \dfrac{5\pi}{32}$

Option c

(10) The set of all odd integers with respect to addition is not a group

Option d

(8) $\lim_{x \to \pi/4} \left(\dfrac{\sqrt{2}\cos x - 1}{\cot x - 1} \right)$

$= \lim_{x \to \pi/4} \dfrac{-\sqrt{2}\sin x}{-\operatorname{cosec}^2 x} = \dfrac{-\sqrt{2}\sin \pi/4}{-\operatorname{cosec}^2 \pi/4}$

$= \dfrac{-\cancel{\sqrt{2}} \cdot 1/\cancel{\sqrt{2}}}{-1/2} = \dfrac{1}{2}$