03/10/2023

① $px^2 + qx + r = 0$

$\alpha + \beta = -q/p$

$\alpha\beta = r/p$

p, q, r are in AP given

$2q = p + r$

$\frac{1}{\alpha} + \frac{1}{\beta} = 4$

$\frac{\alpha+\beta}{\alpha\beta} = 4$

$\frac{-q/p}{r/p} = 4 \Rightarrow \frac{-q}{r} = 4$

$\Rightarrow \boxed{q = -4r}$

$2(-4r) = p + r$

$-8r = p + r$

$\boxed{p = -9r}$

$2q = -9r + r = -8r$

$\boxed{q = -4r}$

$|\alpha - \beta| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$= \sqrt{\frac{q^2}{p^2} - \frac{4r}{p}}$

$= \frac{\sqrt{q - 4pr}}{|p|}$

$= \frac{\sqrt{16r^2 + 36r^2}}{|-9r|}$

$= \frac{\sqrt{(16+36)r^2}}{|-9r|} = \frac{2\sqrt{13}\,\cancel{r}}{9\,\cancel{r}}$

$= \frac{2\sqrt{13}}{9}$

Option b

② $f(n) = \alpha^n + \beta^n$ (given)

$$\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix}$$

$$= \begin{vmatrix} 1+1+1 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \times \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix}^2$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ 1-\alpha & \alpha-\beta & \beta \\ 1-\alpha^2 & \alpha^2-\beta^2 & \beta^2 \end{vmatrix}^2$$

$= [(1-\alpha)(\alpha+\beta)(\alpha-\beta) - (\alpha-\beta)(1+\alpha)(1-\alpha)$

$= [(1-\alpha)(\alpha-\beta)\{\cancel{\alpha}+\beta-1-\cancel{\alpha}\}]^2$

$= [-(1-\alpha)(1-\beta)(\alpha-\beta)]^2$

$= [(1-\alpha)(1-\beta)(\alpha-\beta)]^2$

compare $\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix}$

$= k(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$

$\Rightarrow \boxed{k = 1}$

Option a

(3) Three positive numbers from an increasing G.P. If the middle term in this GP is doubled, the new number are in AP

Let a, ar, ar^2 are in GP

then $a, 2ar, ar^2$ are in AP

$$2 \times 2ar = a + ar^2$$

$$4r = 1 + r^2$$

$$r^2 - 4r + 1 = 0$$

$$r = \frac{4 \pm \sqrt{16-4}}{2}$$

$$r = 2 \pm \sqrt{3}$$

$$r > 1 \Rightarrow \boxed{r = 2 + \sqrt{3}}$$

Option b

(6) $X = 4^n - 3n - 1 = (1+3)^n - 3n - 1$

$$= [{}^nC_0 + {}^nC_1 \cdot 3 + {}^nC_2 \cdot 3^2 + \dots {}^nC_n 3^n] - 3n - 1$$

$$= 9[{}^nC_2 + {}^nC_3 \cdot 3 + \dots + {}^nC_n 3^{n-2}]$$

$\therefore 4^n - 3n - 1$ is a multiple of 9 for all n.

$X = \{x : x \text{ is a multiple of } 9\}$

also, $Y = \{9(n-1) : n \in N\}$

$= \{\text{All multiple of } 9\}$

$$X \subset Y$$

$$\Rightarrow \boxed{X \cup Y = Y}$$

Option b

(4) $$\lim_{x \to 0} \frac{\sin(\pi \cos^2 x)}{x^2}$$

$$= \lim_{x \to 0} \frac{\sin[\pi(1 - \sin^2 x)]}{x^2}$$

$$= \lim_{x \to 0} \frac{\sin(\pi - \pi \sin^2 x)}{x^2}$$

$$= \lim_{x \to 0} \frac{\sin(\pi \sin^2 x)}{x^2}$$

$$= \lim_{x \to 0} \frac{\sin(\pi \sin^2 x)}{\pi \sin^2 x} \cdot \frac{\pi \sin^2 x}{x^2}$$

$$= \pi$$

Option b

(5) If g is the inverse of a function f and $f'(x) = \frac{1}{1+x^5}$

$\Rightarrow f(x)$ and $g(x)$ are inverse of each other

$$\therefore g'[f(x)] = \frac{1}{f'(x)}$$

$$\Rightarrow g'[f(x)] = 1 + x^5$$

$$g'(y) = 1 + [g(y)]^5$$

$$\Rightarrow g'(x) = 1 + (g(x))^5$$

Option b

(7)

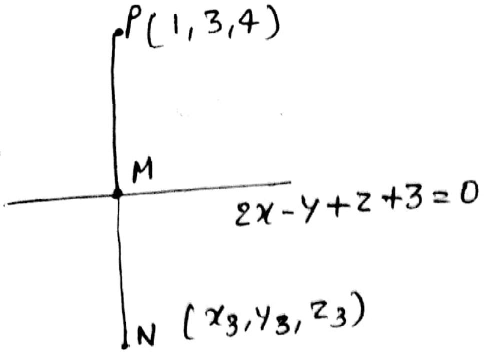


Image N:

$$\frac{x_3-1}{2}=\frac{y_3-3}{-1}=\frac{z_3-4}{1}=\frac{-2(2\cancel{-3}+4\cancel{+3})}{4+1+1}$$

$$\frac{x_3-1}{2}=\frac{y_3-3}{-1}=\frac{z_3-4}{1}=\frac{-(6)\times 2}{6}=-2$$

$\frac{x_3-1}{2}=-2 \Rightarrow x_3=-4+1=-3$

$\frac{y_3-3}{-1}=-2 \Rightarrow y_3=2+3=5$

$\frac{z_3-4}{1}=-2 \Rightarrow z_3=-2+4=2$

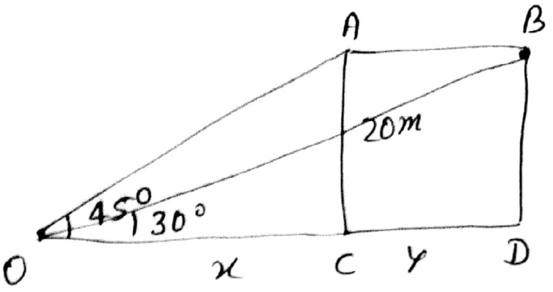
Image $(-3,5,2)$

Equation of line

$$\boxed{\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}}$$

option c

(8)



Time $t=1$ second

$\tan 45°=\frac{20}{x} \Rightarrow \boxed{x=20}$

$\tan 30°=\frac{20}{x+y}$

$\frac{1}{\sqrt{3}}=\frac{20}{20+y}$

$20+y=20\sqrt{3}$

$y=20\sqrt{3}-20$

$=20(\sqrt{3}-1)$

$\boxed{\text{Speed}=20(\sqrt{3}-1)\,m/s}$

option b

(9) $[\bar{a}\times\bar{b}\,.\,\bar{b}\times\bar{c},\ \bar{c}\times\bar{a}]$

$(\bar{a}\times\bar{b})\cdot[(\bar{b}\times\bar{c})\times(\bar{c}\times\bar{a})]$

$=(\bar{a}\times\bar{b})\left([\bar{b}\bar{c}\bar{c}]\bar{a}-[\bar{b}\bar{a}\bar{c}]\bar{c}\right)$

$=[(\bar{a}\times\bar{b})\cdot c][\bar{a}\bar{b}\bar{c}]$

$=[\bar{a}\bar{b}\bar{c}][\bar{a}\bar{b}\bar{c}]$

$=[\bar{a}\bar{b}\bar{c}]^2$

Compare $[\bar{a}\times\bar{b},\ \bar{b}\times\bar{c},\ \bar{c}\times\bar{a}]$

$=\lambda[\bar{a}\bar{b}\bar{c}]^2$

$\Rightarrow \boxed{\lambda=1}$

option b

10 $\quad I = \int \left(1 + x - \frac{1}{x}\right) e^{x+\frac{1}{x}}\, dx$

$$= \int e^{x+\frac{1}{x}}\, dx + \int \left(x - \frac{1}{x}\right) e^{x+\frac{1}{x}}\, dx$$

$$= x e^{x+\frac{1}{x}} - \int x \left(1 - \frac{1}{x^2}\right) e^{x+\frac{1}{x}}\, dx + \int \left(x - \frac{1}{x}\right) e^{x+\frac{1}{x}}\, dx$$

$$= x e^{x+\frac{1}{x}} - \cancel{\int \left(x - \frac{1}{x}\right) e^{x+\frac{1}{x}}\, dx} + \cancel{\int \left(x - \frac{1}{x}\right) e^{x+\frac{1}{x}}\, dx}$$

$$= x\, e^{x+\frac{1}{x}} + C$$

option d