Sol^n 25/10/2023

① $u = \tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

$$u = \tan^{-1}\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}$$

$$= \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)$$

$$= \tan^{-1}\left(\tan\theta/2\right) = \theta/2$$

$$u = \frac{1}{2}\tan^{-1}x$$

$$\frac{du}{dx} = \frac{1}{2}\times\frac{1}{1+x^2}$$

let $v = \tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$

Put $x = \sin\phi \Rightarrow \phi = \sin^{-1}x$

$$v = \tan^{-1}\left(\frac{2\sin\phi\sqrt{1-\sin^2\phi}}{1-2\sin^2\phi}\right)$$

$$= \tan^{-1}\left(\frac{2\sin\phi\cos\phi}{1-2\sin^2\phi}\right)$$

$$= \tan^{-1}\left(\frac{\sin2\phi}{\cos2\phi}\right)$$

$$= \tan^{-1}(\tan2\phi) = 2\phi = 2\sin^{-1}x$$

$$\frac{du}{dv} = \frac{1}{2(1+x^2)}\cdot\frac{\sqrt{1-x^2}}{2}$$

$$\frac{du}{dv} = \frac{\sqrt{1-x^2}}{4(1+x^2)}$$

$$\frac{du}{dv} \text{ at } x=\frac{1}{2} = \frac{\sqrt{1-\frac{1}{4}}}{4(1+\frac{1}{4})}$$

$$= \frac{\sqrt{3}/2}{\cancel{4}\cdot 5/\cancel{4}}$$

$$= \frac{\sqrt{3}}{10}$$

Option d

② $x = 3\tan t$

$$\frac{dx}{dt} = 3\sec^2 t$$

$$y = 3\sec t$$

$$\frac{dy}{dx} = 3\sec t\cdot\tan t$$

$$\frac{dy}{dx} = \frac{3\cancel{\sec t}\cdot\tan t}{3\cancel{\sec t}\sec t}$$

$$= \frac{\sin t}{\cancel{\cos t}}\cdot\cancel{\cos t} = \sin t$$

$$\boxed{\frac{dy}{dx} = \sin t}$$

$$\frac{d^2y}{dx^2} = \cos t\cdot\frac{1}{3\sec^2 t}$$

$$\left.\frac{d^2y}{dx^2}\right|_{t=\pi/4} = \cos\frac{\pi}{4}\cdot\frac{1}{3\sec^2\pi/4}$$

$$= \frac{1}{\sqrt{2}}\cdot\frac{1}{3\cdot 2}$$

$$= \frac{1}{6\sqrt{2}}$$

option b

③ $\frac{d^2x}{dy^2} = \frac{d}{dy}\left(\frac{dx}{dy}\right)$

$$= \frac{d}{dx}\left(\frac{dx}{dy}\right)\cdot\frac{dx}{dy}$$

$$= \frac{d}{dx}\left[\frac{1}{dy/dx}\right]\cdot\frac{1}{dy/dx}$$

$$= -\frac{1}{(dy/dx)^2}\cdot\frac{d^2y}{dx^2}\,\frac{1}{(dy/dx)}$$

$$= -\left(\frac{dy}{dx}\right)^{-3}\frac{d^2y}{dx^2}$$

$$= -\frac{d^2y}{dx^2}\left(\frac{dy}{dx}\right)^{-3}$$

Option d

④ $f(x) = \frac{1}{2}\left(\frac{\cos x}{|\sin x|} + \frac{\sin x}{|\cos x|}\right)$

$f(2\pi + x) = f(x)$

Hence period is 2π

option c

⑤ $f(x) = 3\sin(2x+1)$

Period of $f(x) = \frac{2\pi}{2}$

$= \pi$

option c

⑥ $f(x) = \exp\sqrt{5x-3-2x^2}$

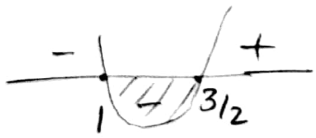
$5x-3-2x^2 \geq 0$

$2x^2 - 5x + 3 \leq 0$

$2x^2 - 2x - 3x + 3 \leq 0$

$2x(x-1) - 3(x-1) \leq 0$

$(2x-3)(x-1) \leq 0$



$\boxed{x \in [1, \tfrac{3}{2}]}$

option b

⑦ $f(x) = \dfrac{1}{\sqrt{|\cos x| + \cos x}}$

$|\cos x| + \cos x > 0$

$\Rightarrow \cos x > 0$

$\Rightarrow -\frac{\pi}{2} < x < \pi/2$

$\Rightarrow 2n\pi - \frac{\pi}{2} < x < 2n\pi + \frac{\pi}{2}$

$\Rightarrow (4n-1)\frac{\pi}{2} < x < (4n+1)\pi/2$

$\Rightarrow x \in \left[\frac{(4n-1)\pi}{2}, \frac{(4n+1)\pi}{2}\right]$

option d

⑧ $f(x) = 2x^3 + mx^2 - 13x + n$

2, 3 are the roots of the equation $f(x) = 0$

$\Rightarrow f(2) = 0$ and $f(3) = 0$

$f(2) = 16 + 4m - 26 + n = 0$

$f(3) = 54 + 9m - 39 + n = 0$

$4m + n - 10 = 0$

$9m + n + 15 = 0$

$-5m - 25 = 0$

$-5m = 25$

$\Rightarrow \boxed{m = -5}$

$-20 + n - 10 = 0$

$\boxed{n = 30}$

$\boxed{m = -5,\ n = 30}$

option c

(9) $A = \{1, 2, 3\}$

$B = \{x, y\}$

Total number of function from A to B

$= 2^3 = 8$

option b

(10) $f: N \to IN$

$f(n) = 2n + 3$

$f'(n) = 2 > 0$

$\Rightarrow f(n)$ is one-one function

Let $y = 2n + 3$

$n = \frac{y-3}{2}$

Hence not an onto function

$\Rightarrow$ injective only

option b