Sol^n 21/10/2023

$y = a^x$ —— ①

$\frac{dy}{dx} = a^x \log a = m_1$

$y = b^x$ —— ②

$\frac{dy}{dx} = b^x \log b = m_2$

$\tan\alpha = \frac{m_1 - m_2}{1 + m_1 m_2}$

$\tan\alpha = \frac{a^x \log a - b^x \log b}{1 + \log a \log b}$

from ① and ②

$a^x = b^x$

$\left(\frac{a}{b}\right)^x = 1$

$\Rightarrow \boxed{x = 0}$

$\tan\alpha = \frac{\log a - \log b}{1 + \log a \log b}$

Option a

② $x^3 - 3xy^2 + 2 = 0$ — ①

$3x^2 - 6yx\frac{dy}{dx} - 3y^2 = 0$

$\frac{dy}{dx} = \frac{x^2 - y^2}{2xy} = m_1$

$3x^2y - y^3 = 2$ — ②

$3x^2\frac{dy}{dx} + 6xy - 3y^2\frac{dy}{dx} = 0$

$(3x^2 - 3y^2)\frac{dy}{dx} = -6xy$

$\frac{dy}{dx} = -\frac{2xy}{x^2 - y^2} = m_2$

Let (x_1, y_1) be the point of intersection then

$m_1 \big|_{(x_1, y_1)} = \frac{x_1^2 - y_1^2}{2x_1y_1}$

$m_2 \big|_{(x_2, y_2)} = -\frac{2x_1y_1}{x_1^2 - y_1^2}$

$m_1 m_2 = -1$

$\Rightarrow$ The curves cut at right angles

$\Rightarrow$ Cut orthogonally

Option a

③ $y^2 = 2x$ — ①

$2y\frac{dy}{dx} = 2 \Rightarrow \frac{dy}{dx} = \frac{1}{y} = m_1$

$2xy = K$ — ②

$2x\frac{dy}{dx} + 2y = 0$

$\frac{dy}{dx} = -\frac{y}{x} = m_2$

Solving ① and ②

$y^2 = \frac{K}{y} \Rightarrow y^3 = K$

$\Rightarrow y = K^{1/3}$

$x = \frac{K^{2/3}}{2}$

Point $\left(\frac{K^{2/3}}{2}, K^{1/3}\right)$

$m_1 \big|_{\left(\frac{K^{2/3}}{2}, K^{1/3}\right)} = \frac{1}{K^{1/3}}$

$m_2 \big|_{\left(\frac{K^{2/3}}{2}, K^{1/3}\right)} = -\frac{2K^{1/3}}{K^{2/3}}$

given cut at right angle

$m_1 m_2 = -1$

$\frac{1}{K^{1/3}} \cdot \frac{-2K^{1/3}}{K^{2/3}} = -1$

$2 = K^{2/3} \Rightarrow K^2 = 8$

Option a

④ $\log_{x_1}\log_{x_2}\log_{x_3}\cdots\log_{x_n} x_n^{x_{n-1}^{\cdots^{x_1}}}$

$= \log_{x_1}\log_{x_2}\log_{x_3}\cdots\log_{x_{n-1}}\left(x_{n-1}^{x_{n-2}^{\cdots^{x_1}}}\log x_n^{x_n}\right)$

$= \log_{x_1}\log_{x_2}\log_{x_3}\cdots\log_{x_{n-1}} x_{n-1}^{x_{n-2}^{\cdots^{x_1}}}$

$= \cdots\cdots = \log_{x_1}^{x_1} = 1$

option b

⑤ $\log_4(x-1) = \log_2(x-3)$

$\log_{2^2}(x-1) = \log_2(x-3)$

$\frac{1}{2}\log_2(x-1) = \log_2(x-3)$

$\log_2(x-1)^{1/2} = \log_2(x-3)$

$(x-1)^{1/2} = x-3$

$(x-1) = (x-3)^2$

$x-1 = x^2-6x+9$

$x^2-7x+10 = 0$

$x^2-5x-2x+10=0$

$x(x-5)-2(x-5)=0$

$(x-5)(x-2)=0$

$x = 2, 5$

$x = 5$

⑥ $f(x) = \frac{2^x+2^{-x}}{2}$

$f(x+y)\,f(x-y)$

$= \frac{2^{x+y}+2^{-x-y}}{2}\cdot\frac{2^{x-y}+2^{-x+y}}{2}$

$= \frac{1}{4}\left(2^{x+y}+2^{-x-y}\right)\left(2^{x-y}+2^{-x+y}\right)$

$= \frac{1}{4}\left[2^{x+y}\cdot 2^{x-y}+2^{x+y}\cdot 2^{-x+y} + 2^{-x-y}\cdot 2^{x-y} + 2^{-x-y}\cdot 2^{-x+y}\right]$

$= \frac{1}{4}\left[2^{2x}+2^{2y}+2^{-2y}+2^{-2x}\right]$

$= \frac{1}{4}\left[2^{2x}+2^{-2x}+2^{2y}+2^{-2y}\right]$

$= \left[\frac{2^{2x}+2^{-2x}}{2}+\frac{2^{2y}+2^{-2y}}{2}\right]\cdot\frac{1}{2}$

$= \frac{1}{2}\left[f(2x)+f(2y)\right]$

option b

$$y = \frac{a^x - a^{-x}}{a^x + a^{-x}}$$

$$y = \frac{a^{2x} - 1}{a^{2x}+1}$$

$$y(a^{2x}+1) = a^{2x} - 1$$

$$ya^{2x} - a^{2x} = -1 - y$$

$$(y-1)\,a^{2x} = -1-y$$

$$a^{2x} = \frac{-1-y}{y-1}$$

$$a^{2x} = \frac{1+y}{1-y}$$

$$x = \frac{1}{2}\log_a\left(\frac{1+y}{1-y}\right)$$

$$f^{-1}(y) = \frac{1}{2}\log\left(\frac{1+y}{1-y}\right)$$

$$f^{-1}(x) = \frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$$

option c

(8)

$$\lim_{x\to 1} \frac{ab^x - a^x b}{x-1}$$

$$= \lim_{x\to 1} \frac{ab^x \log b - (a^x \log a)\,b}{1}$$

$$= ab\log b - ab\log a$$

$$= ab(\log b - \log a)$$

$$= ab\log\left(\frac{b}{a}\right)$$

option c

(9)

$$R^2 = P^2 + Q^2 + 2PQ\cos\alpha$$

$$S^2 = P^2 + (-Q)^2 + 2P(-Q)\cos\alpha$$

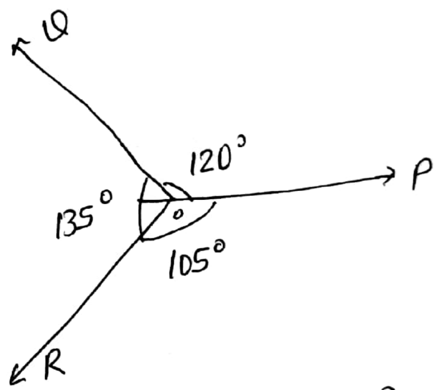
$$= P^2 + Q^2 - 2PQ\cos\alpha$$

$$R^2 + S^2 = 2(P^2+Q^2)$$

option b

(10) By Lami's Theorem

$$\frac{P}{\sin 135^\circ} = \frac{Q}{\sin 105^\circ} = \frac{R}{\sin 120^\circ}$$



$$\frac{P}{1/\sqrt{2}} = \frac{Q}{\frac{\sqrt{3}+1}{2\sqrt{2}}} = \frac{R}{\frac{\sqrt{3}}{2}}$$

$$\frac{\sqrt{2}P}{1} = \frac{2\sqrt{2}Q}{\sqrt{3}+1} = \frac{2R}{\sqrt{3}}$$

$$\frac{P}{1} = \frac{2Q}{\sqrt{3}+1} = \frac{\sqrt{2}R}{\sqrt{3}}$$

$$\frac{P}{2} = \frac{Q}{\sqrt{3}+1} = \frac{R}{\sqrt{6}}$$

$$P : Q : R = 2 : \sqrt{3}+1 : \sqrt{6}$$

option b