Solution - 19/10/202

(1) $\lim_{x\to 0} \dfrac{xe^x - \log(1+x)}{x^2}$

$= \lim_{x\to 0} \dfrac{xe^x + e^x - \frac{1}{1+x}}{2x}$

$= \lim_{x\to 0} \dfrac{xe^x + e^x + e^x + \frac{1}{(1+x)^2}}{2}$

$= \dfrac{3}{2}$ Option a

(2) $\lim_{x\to 0} \dfrac{\log(1+x^3)}{\sin^3 x}$

$= \lim_{x\to 0} \dfrac{\frac{\cancel{3}x^2}{1+x^3}}{\cancel{3}\sin^2 x \cos x}$

$= \lim_{x\to 0} \left(\dfrac{x}{\sin x}\right)^2 \dfrac{1}{\cos x\,(1+x^3)}$

$= 1$ Option a

(3) $\lim_{x\to \frac{1}{2}} \dfrac{\cos^2 \pi x}{e^{2x} - 2xe}$

$= \lim_{x\to \frac{1}{2}} \dfrac{\cancel{2}(\cos \pi x)(-\sin \pi x)\cdot\pi}{\cancel{2}\cdot e^{2x} - \cancel{2}e}$

$= \lim_{x\to \frac{1}{2}} \dfrac{\pi(\cos \pi x)\cdot(-\pi)\cos \pi x + \pi(-\sin \pi x)\cdot\pi(-\sin \pi x)}{2e^{2x}}$

$= \dfrac{\pi^2}{2e}$ Option a

(4) $\lim_{x\to y} \dfrac{x^y - y^x}{x^x - y^y}$

$= \lim_{x\to y} \dfrac{yx^{y-1} - y^x \log y}{(1+\log x)x^x - 0}$

$= \dfrac{yy^{y-1} - y^y \log y}{y^y(1+\log y)}$

$= \dfrac{1-\log y}{1+\log y}$

Option b

(5) $\lim_{x\to 0} \dfrac{\sqrt{x}\tan x}{(e^x-1)^{3/2}}$

$= \lim_{x\to 0} \dfrac{x\sqrt{x}}{(e^x-1)^{3/2}} \cdot \dfrac{\tan x}{x}$

$= \lim_{x\to 0} \dfrac{x^{3/2}}{(e^x-1)^{3/2}}$

$= \lim_{x\to 0} \left(\dfrac{x}{e^x-1}\right)^{3/2}$

$= (1)^{3/2}$

$= 1$

Option c

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(6) $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$

$\cos u = \frac{x+y}{\sqrt{x}+\sqrt{y}}$

$n = \frac{1}{2}$

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$

$x\frac{\partial}{\partial x}(\cos u) + y\frac{\partial}{\partial y}\cos u = \frac{1}{2}\cos u$

$-x\sin u\frac{\partial u}{\partial x} + y(-\sin u)\frac{\partial u}{\partial y} = \frac{1}{2}\cos u$

$\sin u \cdot x\frac{\partial u}{\partial x} + \sin u \cdot y\frac{\partial u}{\partial y} = -\frac{1}{2}\cos u$

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -\frac{1}{2}\frac{\cos u}{\sin u}$

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0$

option d

(7) $u = \sin^{-1}\frac{x^2+y^2}{x+y}$

$\sin u = \frac{x^2+y^2}{x+y}$ $\quad n = 1$

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$

$x\frac{\partial}{\partial x}(\sin u) + y\frac{\partial}{\partial y}(\sin u) = 1 \cdot \sin u$

$x\frac{\partial u}{\partial x}\cdot\cos u + y\cos u\frac{\partial u}{\partial y} = \sin u$

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$

option c

(8) $u = \log_e\left(\frac{x^2+y^2}{x+y}\right)$

$e^u = \frac{x^2+y^2}{x+y}$ $\quad n = 1$

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n\frac{f(u)}{f'(u)} = 1\cdot\frac{e^u}{e^u}$

$$\boxed{x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1}$$

$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2}$

$= g(u)\,[g'(u) - 1]$

$= 1\,(0-1)$

$= -1$

$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -1$

option b

(9) $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$

$\tan u = \frac{x^3+y^3}{x-y}$ $\quad n = 2$

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n\frac{f(u)}{f'(u)}$

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\cdot\frac{\tan u}{\sec^2 u}$

$= 2\frac{\sin u \cos u \cancel{\cos u}}{\cancel{\cos u}}$

$= \sin 2u$

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u. = [g(u)]$

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}$$

$$=g(u)[g'(u)-1]$$

$$=\sin 2u\left[2\cos 2u-1\right]$$

$$=2\sin 2u\cos 2u-\sin 2u$$

$$=\sin 4u-\sin 2u$$

$$=\sin 2u\left[2\cos 2u-1\right]$$

$$=\sin 2u\left[2(1-2\sin^2 u)-1\right]$$

$$=\sin 2u\left[2-4\sin^2 u-1\right]$$

$$=\sin 2u\left[1-4\sin^2 u\right]$$

option a

(10) $z=x^4y^2\sin^{-1}\frac{x}{y}+\log x-\log y$

Let $u=x^4y^4\sin^{-1}\frac{x}{y}$

$v=\log x-\log y$

$v=\log\frac{x}{y}$

$z=u+v$

$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=6u$ and

$x\frac{\partial v}{\partial x}+y\frac{\partial v}{\partial y}=0$

$$\boxed{x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0}$$

$$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=6u+0$$

$$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=6\cdot x^4y^4\sin^{-1}\frac{x}{y}$$

option b