(1) $A = \{1, 2, 3\}$ $B = \{a, b\}$

$A \times B = \{(1,a), (1,b), (2,a)\ (2,b)$
$(3,a)\ (3,b)\}$

Option C

(2) $x^2 + a^2 = 1 - 2ax$ — (1)
$x^2 + b^2 = 1 - 2bx$ — (2)

Let β be the common root of both the quadratic equations

$\beta^2 + a^2 = 1 - 2a\beta$
$\beta^2 + 2a\beta + a^2 - 1 = 0$ — (3)
$\beta^2 + 2b\beta + b^2 - 1 = 0$ — (4)

$$\frac{\beta^2}{2a(b^2-1) - 2b(a^2-1)} = \frac{-\beta}{(b^2-1) - (a^2-1)}$$
$$= \frac{1}{2b - 2a}$$

$$\frac{\beta^2}{2ab^2 - 2ba^2 - 2a + 2b} = \frac{\beta}{(a^2-1) - (b^2-1)}$$
$$= \frac{1}{2b - 2a}$$

$$\frac{\beta^2}{2ba\ (b-a) + 2(b-a)} = \frac{\beta}{(a^2-1) - (b^2-1)}$$
$$= \frac{1}{2(b-a)}$$

$$\frac{\beta^2}{2(b-a)[ab+1]} = \frac{\beta}{-(b-a)(a+b)} = \frac{1}{2(b-a)}$$

$$\beta = -\frac{a+b}{2}$$

$\beta^2 = ab + 1$

$$ab + 1 = \left(-\frac{a+b}{2}\right)^2$$

$4(ab+1) = a^2 + b^2 + 2ab$
$4ab + 4 = a^2 + b^2 + 2ab$
$a^2 + b^2 - 2ab = 4$
$(a-b)^2 = 4$
$\boxed{a - b = 2}$

Option C

(3) PENCIL

Total number of alphabet $= 6$
Consonants $= 4$

C C C C

Remaining two places can be filled by two vowels in $5P_2$ ways

The total number of ways $= 4! \times 5P_2$
$= 24 \times 20$
$= 480$

Option d

(4) $I \int_0^2 \frac{dx}{1+5x}$

Put $1 + 5x = t$
$5dx = dt$
$dx = \frac{1}{5} dt$

$$= \frac{1}{5}\int_1^{11} \frac{1}{t}\,dt$$

$$= \frac{1}{5} \log t \Big|_1^{11}$$

$$= \frac{1}{5} \log 11$$

option d

(5) The number of signals that can sent by 5 flags of different colours taking one or more at a time

$$5P_1 + 5P_2 + 5P_3 + 5P_4 + 5P_5$$

$$= 5 + 20 + 60 + 120 + 120$$

$$= 325$$

option d

(6) $(1+x)^{30} = 1 + 30C_1 + 30C_2 x^2 + 30C_3 x^3 + \cdots + 30C_{29} x^{29} + 30C_{30} x^{30}$

sum of coefficients of odd powers of x is

$$30C_1 + 30C_3 + \cdots 30C_{29}$$

$$= 2^{30-1}$$

$$= 2^{29}$$

option d

(7) $\left[\sqrt{\frac{x}{3}} + \frac{\sqrt{3}}{x^2}\right]^{10}$

$$T_{r+1} = 10C_r \left(\frac{x}{3}\right)^{\frac{10-r}{2}} \left(\frac{(3)^{1/2}}{x^2}\right)^r$$

$$= 10C_r \left(\frac{1}{3}\right)^{\frac{10-r}{2}} \cdot 3^{r/2} \, x^{\frac{10-r}{2} - 2r}$$

for term independent of x we have

$$\frac{10-r}{2} - 2r = 0$$

$$10 - r - 4r = 0$$

$$5r = 10$$

$$\boxed{r = 2}$$

Term independent of x

$$= 10C_2 \left(\frac{1}{3}\right)^{\frac{10-2}{2}} (3)^{2/2}$$

$$= 10C_2 \left(\frac{1}{3}\right)^4 \times 3$$

$$= \frac{\cancel{10}^{5} \times 9}{\cancel{2}} \cdot \frac{1}{3^3} = \frac{45}{27} = \frac{5}{3}$$

option a

9

$$\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix} = \begin{vmatrix} \frac{\log x}{\log x} & \frac{\log y}{\log x} & \frac{\log z}{\log x} \\ \frac{\log x}{\log y} & \frac{\log y}{\log y} & \frac{\log z}{\log y} \\ \frac{\log x}{\log z} & \frac{\log y}{\log z} & \frac{\log z}{\log z} \end{vmatrix}$$

$$= \frac{1}{\log x \log y \log z} \begin{vmatrix} \log x & \log y & \log z \\ \log x & \log y & \log z \\ \log x & \log y & \log z \end{vmatrix}$$

$$= 0$$

option d

8

$$\begin{vmatrix} y+z & x & x \\ y & z+x & y \\ z & z & x+y \end{vmatrix}$$

$R_2 \to R_2 - R_1$

$$= \begin{vmatrix} y+z & x & x \\ -z & z & y-x \\ z & z & x+y \end{vmatrix}$$

$$= \begin{vmatrix} x+y+z & x & x \\ 0 & z & y-x \\ 2z & z & x+y \end{vmatrix}$$

$$= (x+y+z)(xz + \cancel{yz} - \cancel{yz} + xz)$$

$$+ 2z(xy - x^2 - xz)$$

$$= \cancel{2x^2z} + 2xyz + \cancel{2xz^2}$$

$$+ 2xyz - \cancel{2x^2z} - \cancel{2xz^2}$$

$$= 4xyz$$

option c

10 $\frac{1^2}{3!} + \frac{2^2}{4!} + \frac{3^2}{5!} + \cdots$

$$T_n = \frac{n^2}{(n+2)!}$$

$$= \frac{(n+2)n - 2(n+2) + 4}{(n+2)!}$$

$$= \frac{n(n+2)}{(n+2)!} - \frac{2(n+2)}{(n+2)!} + \frac{4}{(n+2)!}$$

$$= \frac{(n+1)-1}{(n+1)!} - \frac{2}{(n+1)!} + \frac{4}{(n+2)!}$$

$$= \frac{1}{n!} - \frac{1}{(n+1)!} - \frac{2}{(n+1)!} + \frac{4}{(n+2)!}$$

$$= \frac{1}{n!} - \frac{3}{(n+1)!} + \frac{4}{(n+2)!}$$

$$S_n = \sum_{n=1}^{\infty} \frac{1}{n!} - \sum_{n=1}^{\infty} \frac{3}{(n+1)!} + \sum_{n=1}^{\infty} \frac{4}{(n+2)!}$$

$$= (e-1) - 3(e-2) + 4(e - \tfrac{5}{2})$$

$$= e - 3e + 4e - 1 + 6 - 10$$

$$= 2e - 5$$

option c