Soln. 01/10/2023

① $\left(x+\frac{1}{x}\right)^{10}$

$n = 10$ even

middle term $= \left(\frac{10}{2}+1\right)^{th}$ term

$= 6^{th}$ term

$T_6 = {}^{10}C_5 \cdot \cancel{x^5} \cdot \frac{1}{\cancel{x^5}}$

$= {}^{10}C_5$ option b

② $\left(x^2 - \frac{1}{x}\right)^9$

$T_{r+1} = {}^9C_r\,(x^2)^{9-r}\left(-\frac{1}{x}\right)^r$

$= {}^9C_r\,(-1)^r\,(x)^{18-2r-r}$

$= {}^9C_r\,(-1)^r\,(x)^{18-3r}$

For constant term

$18-3r=0$

$r=6$

Constant term $= {}^9C_6\,(-1)^6$

$= \frac{3\cancel{9}\times\cancel{8}^4\times 7}{\cancel{6}_2}$

$= 84$

option c

③ $\left(x+\frac{2}{x^2}\right)^{15}$

$T_{r+1} = {}^{15}C_r\,(x)^{15-r}\left(\frac{2}{x^2}\right)^r$

$= {}^{15}C_r\,(x)^{15-r}\,(2)^r\,x^{-2r}$

$= {}^{15}C_r\,(x)^{15-3r}\,2^r$

For term independent of x $\quad 15-3r=0 \Rightarrow r=5$

The term independent of $x = {}^{15}C_5\,2^5$

option b

④ $\left(x-\frac{1}{x}\right)^7$

$T_{r+1} = {}^7C_r\,(x)^{7-r}\left(-\frac{1}{x}\right)^r$

$= {}^7C_r\,(-1)^r\,(x)^{7-2r}$

For the coefficient of x^3

$7-2r=3$

$4=2r \Rightarrow r=2$

The coefficient of x^3

$= {}^7C_2\,(-1)^2$

$= \frac{7\times\cancel{6}^3}{\cancel{2}} = 21$

option b

⑤ The sum of the coefficients of $(x^2-x-1)^{99}$

$= (\cancel{1}-\cancel{1}-1)^{99}$

$= (-1)^{99}$

$= -1$

option c

⑥ $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a-a & b-b & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \text{unit Matrix}$$

option a

(7)

$$x + 2y + 3z = 1$$
$$x - y + 4z = 0$$
$$2x + y + 7z = 1$$

$$A^* = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 1 & -1 & 4 & 0 \\ 2 & 1 & 7 & 1 \end{array}\right]$$

$R_3 \to R_3 - 2R_1, \quad R_2 \to R_2 - R_1$

$$A^* = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -3 & 1 & -1 \\ 0 & -3 & 1 & -1 \end{array}\right]$$

$$A^* = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -3 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\rho(A^*) = \rho(A) = 2 < n.$$

$\Rightarrow$ System has infinite many solution.

option d

(8)

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$$

$$= abc \begin{vmatrix} 1+1/a & 1/a & 1/a \\ 1/b & 1+1/b & 1/b \\ 1/c & 1/c & 1+1/c \end{vmatrix}$$

$$= abc \begin{vmatrix} 1 & 0 & 1/a \\ 0 & 1 & 1/b \\ -1 & -1 & 1+1/c \end{vmatrix}$$

$$= abc \left[1\left(1 + \frac{1}{c} + \frac{1}{b}\right) + \frac{1}{a}(0+1) \right]$$

$$= abc \left[1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right]$$

given. $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = 0$

$$abc\left[1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right] = 0$$

$$1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$$

$$\Rightarrow \boxed{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = -1}$$

option d

(9)

$$2x + y - 4 = 0 \quad \text{--- (1)}$$
$$x + 2y - 2 = 0 \quad \text{--- (2)}$$
$$3x + 5y - 6 = 0 \quad \text{--- (3)}$$

from (1) and (2) $y = 0, \ x = 2$

which satisfied the eq^n (3) also.

<u>only one solution</u>

option b

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(10) $a_1\ a_2 \ldots\ldots$ form a G.P.

$$\Delta = \begin{vmatrix} \log a_m & \log a_{m+1} & \log a_{m+2} \\ \log a_{m+3} & \log a_{m+4} & \log a_{m+5} \\ \log a_{m+6} & \log a_{m+7} & \log a_{m+8} \end{vmatrix}$$

$$a_{m+1}^2 = a_m a_{m+2}$$

Taking log both side

$$2 \log a_{m+1} = \log a_m + \log a_{m+2}$$

$$2 \log a_{m+4} = \log a_{m+3} + \log a_{m+5}$$

$$2 \log a_{m+7} = \log a_{m+6} + \log a_{m+8}$$

$$\Delta = \frac{1}{2} \begin{vmatrix} \log a_m & \log a_m + \log a_{m+2} & \log a_{m+2} \\ \log a_{m+3} & \log a_{m+3} + \log a_{m+5} & \log a_{m+5} \\ \log a_{m+6} & \log a_{m+6} + \log a_{m+8} & \log a_{m+8} \end{vmatrix}$$

$$C_2 \to C_2 - (C_1 + C_3)$$

$$= \frac{1}{2} \times 0$$

$$\boxed{\Delta = 0}$$

option c